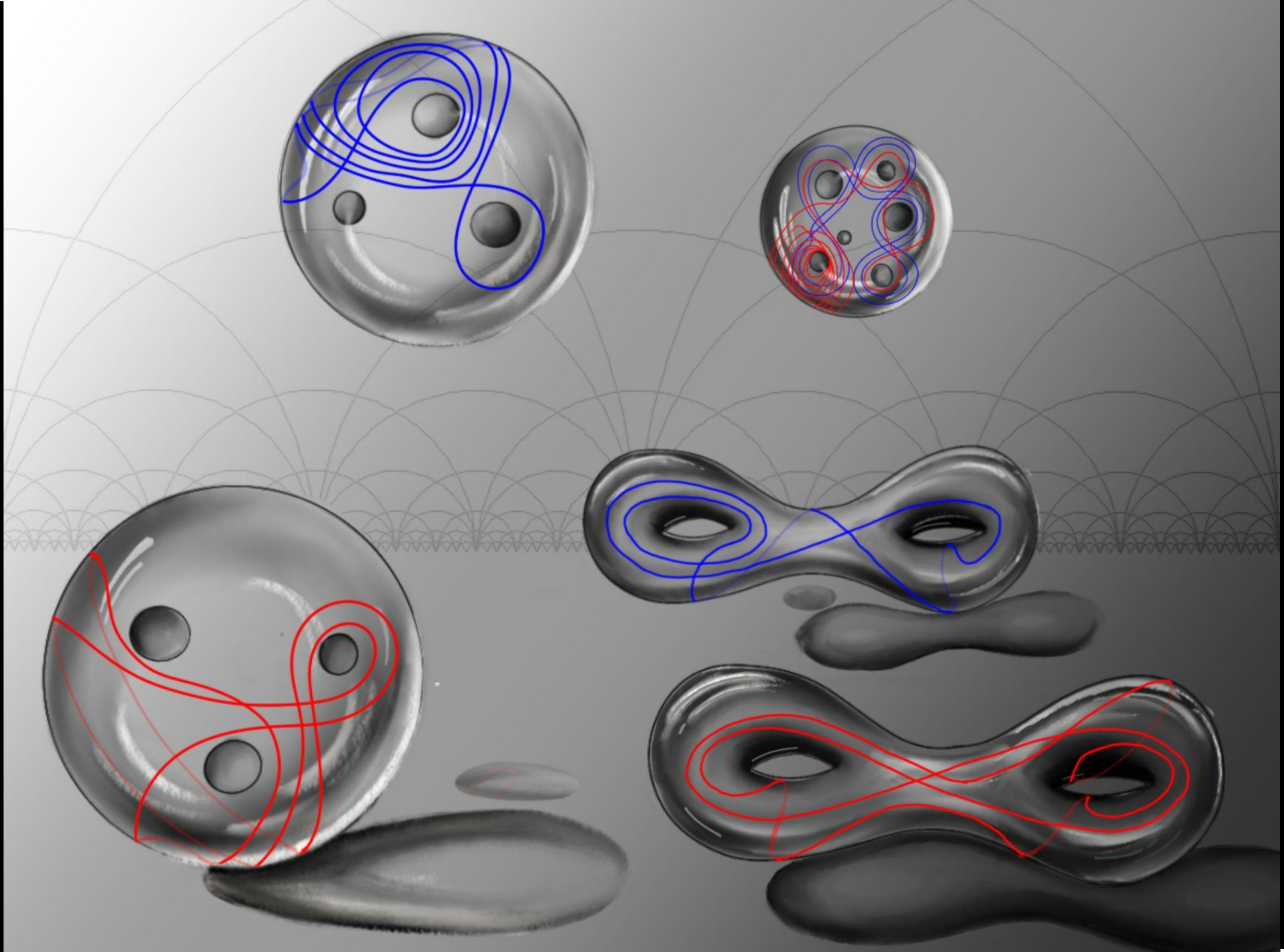

Length infima of non-simple closed curves

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Work in Progress

01
BACKGROUND

02
MOTIVATION

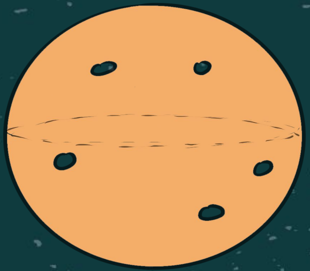
03
QUESTIONS

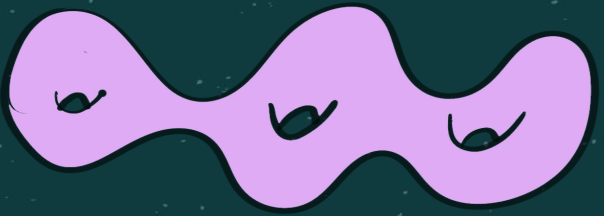
04
MAIN RESULTS

Background

Surfaces

We consider hyperbolic surfaces (Surfaces with negative Euler characteristic)

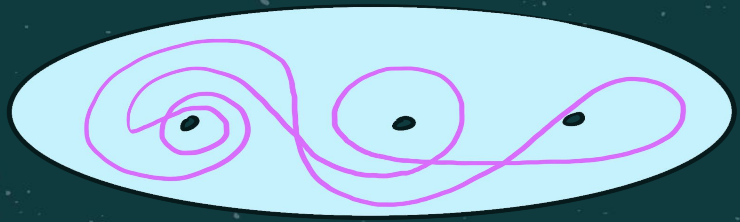




Filling curves

A closed curve on a surface is said to be filling if it intersects every non-trivial simple closed curve on the surface.

Example:



Complement of a filling curve is a union of discs and annuli

Teichmuller space

Set of isotopy classes of marked hyperbolic structures.

Each point in Teichmuller space of Σ, can be denoted as (ϕ, X) where X is a surface with complete, finite area hyperbolic metric with totally geodesic boundary and ϕ is a diffeomorphism from Σ to X.

Length infinimum

Fix a topological surface Σ and let Teich(Σ) denote its Teichmuller space. Consider a non-simple closed curve γ in Σ.

For (ϕ, X) in(Σ) . Let $l_\gamma(X)$ denote the 'X-length' of the geodesic in the free homotopy class of $\phi(\gamma)$.

We define the length infimum of γ as follows:

$$m(\gamma) = \inf\{l_\gamma(X) : (\phi, X) \text{ in Teich}(\Sigma)\}$$

Properties:

- Invariant under action on Mapping Class Group on Teichmuller space.
- Infimum is attained i.e. it is the minimum.
- Unique.

Self intersection number vs Length

Want to buy:

Self intersections

Currency:

Length

Questions

- Does there exist distinct filling curves with same length infimum?

  -Yes

- Does there exist distinct filling curves with same self intersection number that have same length infimum?

  -(Mostly yes)

- Does there exist two distinct filling curves with same self intersection number have different length infimum?

RESULTS

Theorem :

Consider a genus 0 surface with n punctures. For any k (suitably large) we can find two distinct curves with self intersection number k and different length infinimum..

Key Ideas...

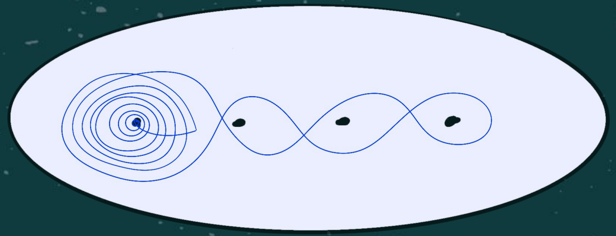
01 Thick vs thin part

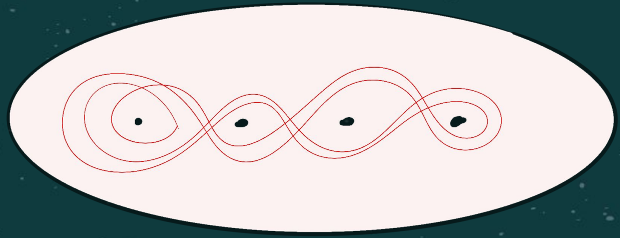
02 Collar lemmas

03 Length bounds near cusps

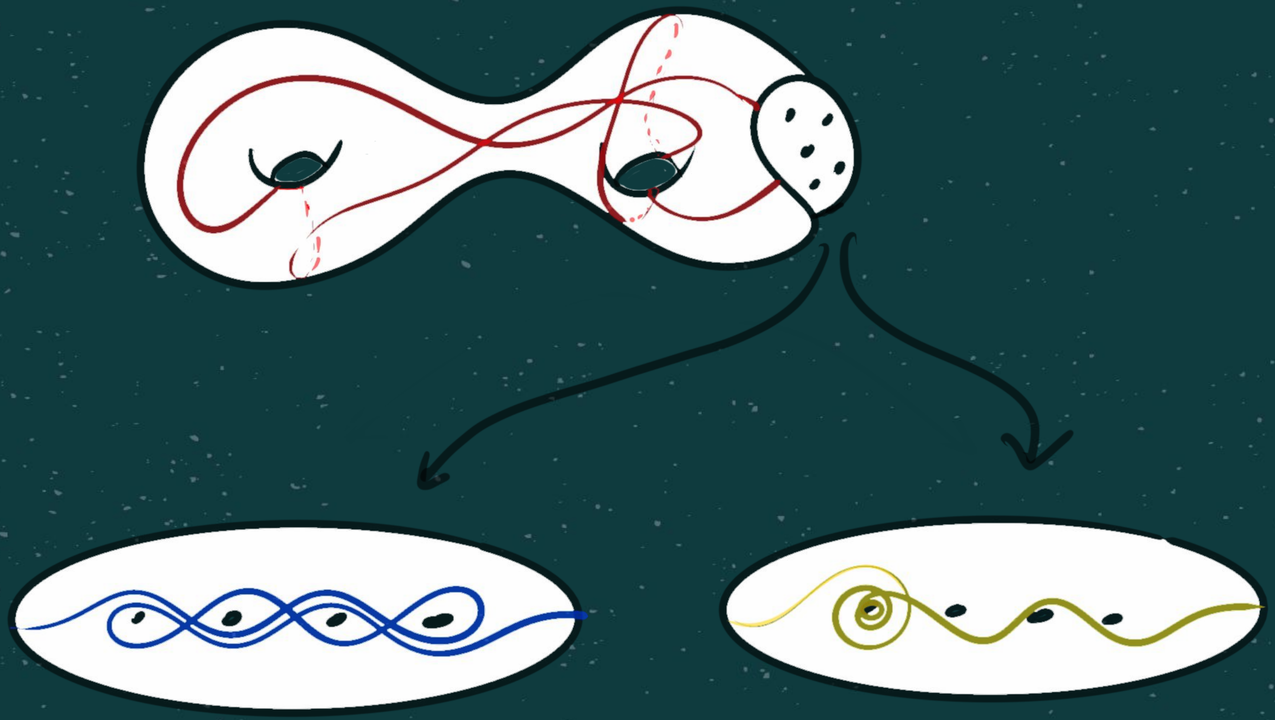
04 Minimal filling curves

Proof (sphere with punctures)

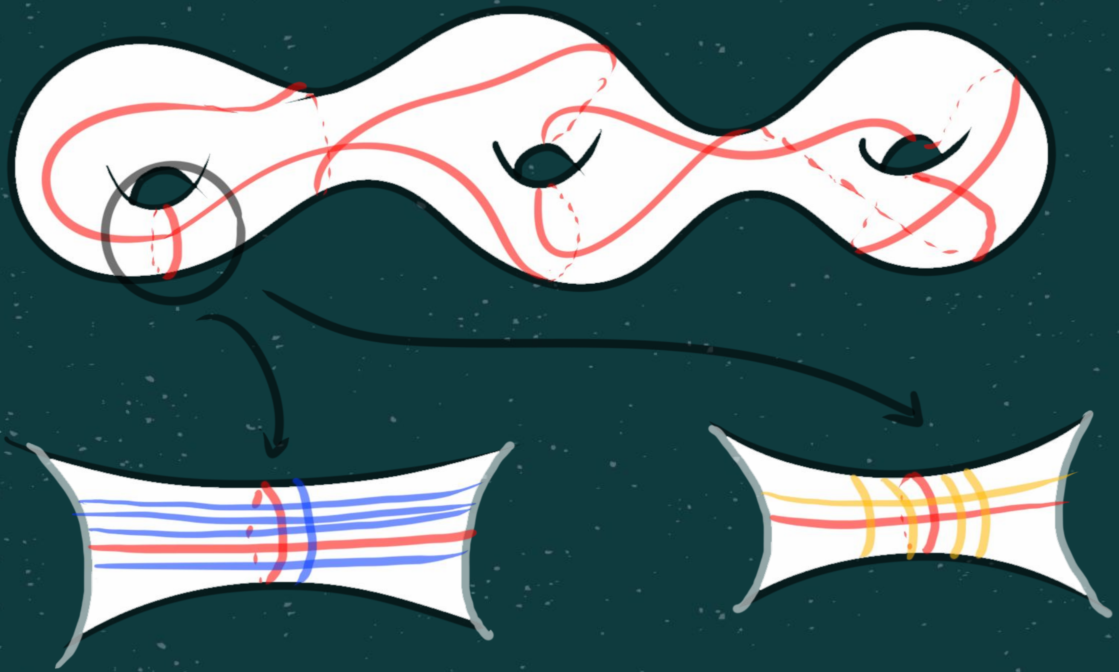




Surfaces with punctures



Closed surfaces



THANK YOU!