Contact structures to foliations and back

Sayantika Mondal

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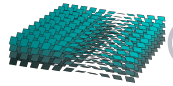
Contact structures



Contact structure

A contact structure is a nowhere integrable plane field on a manifold.

Definition 1 (Contact Structure)

A (oriented) contact structure ξ on a 3 manifold M is an oriented plane $\xi \subset TM$ for which there is a 1-form α such that $\xi = \ker(\alpha)$ and $\alpha \wedge d\alpha > 0$.

Contact structure on R^3

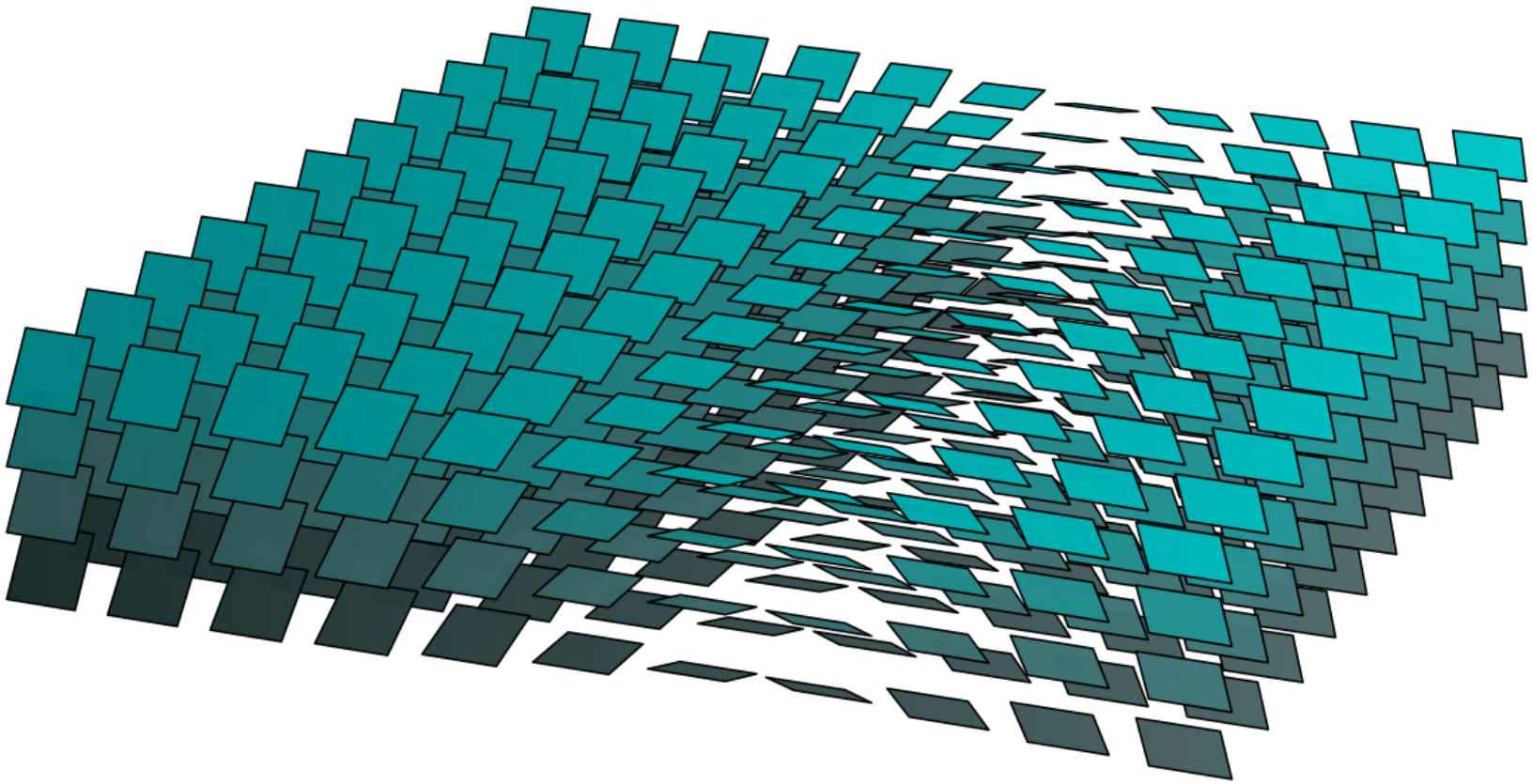
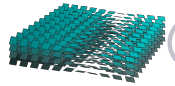


Figure: Standard contact structure on R^3

Foliations



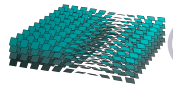
Co-dimension 1 foliation

A co-dimension 1 foliation of the 3-manifold is an integrable plane field.

Definition 1 (Co-dim 1 foliation)

A co-dimension 1 foliation ξ on a 3 manifold M is a plane field given by the kernel of a 1-form α such that $\xi = \ker(\alpha)$ and $\alpha \wedge d\alpha = 0$.

Contact structures and foliations



Theorem (Eliashberg and Thurston)

Foliations can be approximated by contact structures on a closed oriented 3-manifold.

Question

Is every contact structure on a 3-manifold "close" to a foliation?

- Close here means being a deformation.
- A contact structure ξ is said to be a deformation of a foliation ζ if there is a one parameter family of plane fields ξ_t such that $\xi_0 = \zeta$ and $\xi_1 = \xi$, and ξ_t is a contact structure for $t > 0$.

Theorem (Etnyre)

Every positive and negative contact structure on a closed oriented 3-manifold is a C^∞-deformations of a C^∞-foliations.

Tight vs Taut

- There are two types of contact structures Tight and Overtwisted. A contact structure is said to be tight if it has no overtwisted disk.
- A foliation is said to be taut if each leaf is intersected by a transversal closed curve. A taut foliation cannot have Reeb components (necessary but not sufficient).

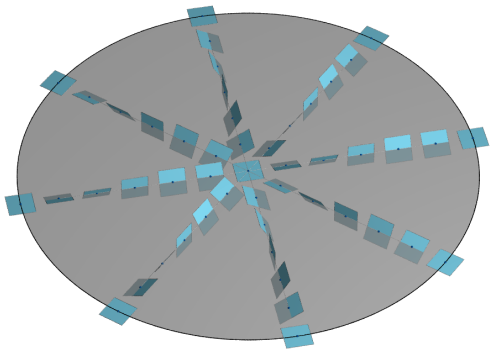


Figure: Overtwisted disk

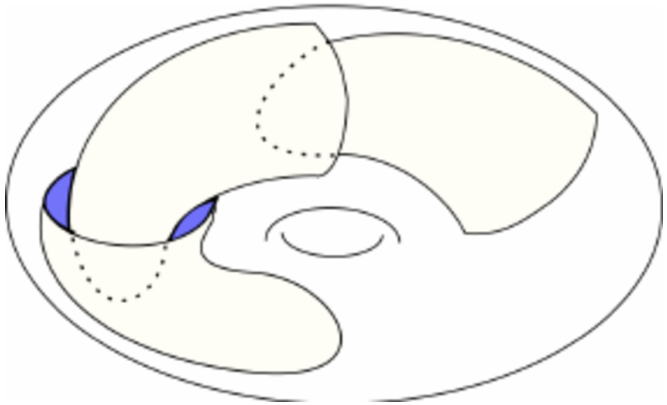
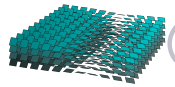


Figure: Reeb component

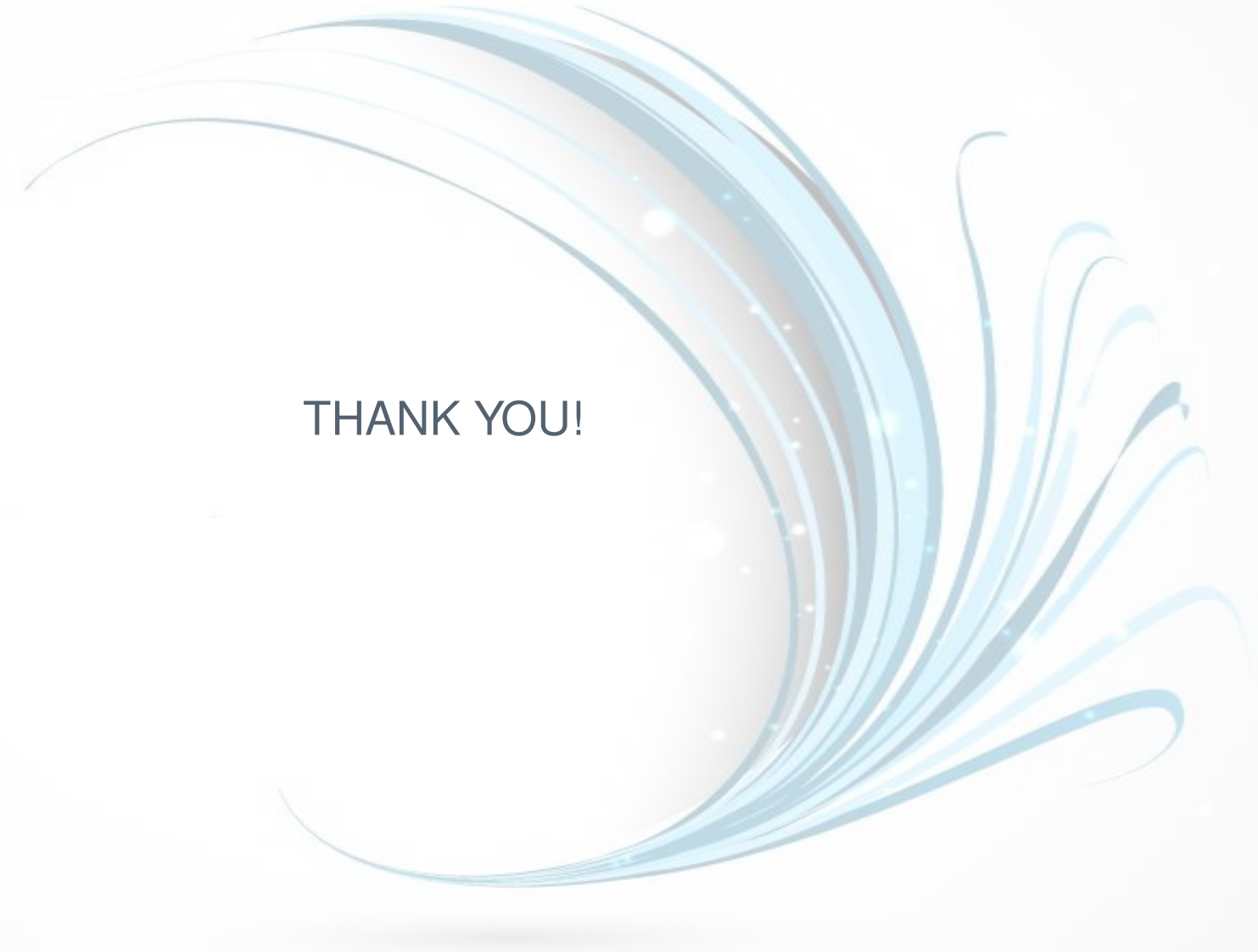
Contact structures and foliations



The foliation constructed in the proof of Etnyre has a Reeb component.

Open Questions

Is every tight contact structure a deformation of a taut / Reebless foliation?


THANK YOU!